Answers to Exercise 1

i. $f(x) = 3x^2 - 4$

$f'(x) = 6x$

ii. $f(x) = 7x^3 - 2x^2 + 5x + 1$

$f'(x) = 21x^2 - 4x + 5$

iii. $f(x) = \frac{2}{3}x^6 + \frac{1}{6}x^{-3} - \frac{1}{2}$

$f'(x) = 4x^5 - \frac{1}{2}x^{-4}$

iv. $f(x) = ax^3 + b$

$f'(x) = 3ax^2$

v. $y = \frac{5}{x}$

$y = 5x^{-1} \rightarrow y' = -5x^{-2} \rightarrow y' = -\frac{5}{x^2}$

vi. $y = \frac{7}{x^4}$

$y = 7x^{-4} \rightarrow y' = -28x^{-5} \rightarrow y' = -\frac{28}{x^5}$

vii. $f(x) = 4ax^2 - 3bx^{-2} + a^2$

$f'(x) = 8ax + 6bx^{-3}$

viii. $f(x) = (4x - 2)x^3$

$f'(x) = 4x^3 + 3x^2(4x - 2) = 4x^3 + 12x^3 - 6x^2 = 16x^3 - 6x^2$

Or

$f(x) = 4x^4 - 2x^3$

$f'(x) = 16x^3 - 6x^2$

ix. $y = (x + \frac{a}{x}) \cdot (x - \frac{a}{x})$

$y = (x + ax^{-1}) \cdot (x - ax^{-1})$

$y' = (1 - ax^{-2}) \cdot (x - ax^{-1}) + (x + ax^{-1}) \cdot (1 + ax^{-2}) \rightarrow (1 - \frac{a}{x^2}) \cdot (x - \frac{a}{x}) + (x + \frac{a}{x}) \cdot (1 + \frac{a}{x^2})$

x. $y = \frac{(2x - 1) \cdot (x + 2)}{x}$

$y = \frac{(2x^2 - x + 4x - 2)}{x} \rightarrow y = 2x - 1 + 4 - \frac{2}{x} \rightarrow y = 2x + 3 - \frac{2}{x} \rightarrow y = 2x + 3 - 2x^{-1}$

$$y' = 2 + 2x^{-2} \rightarrow y' = 2 + \frac{2}{x^2}$$

xi. $y = \frac{x - a}{bx^3}$

$$y' = \frac{bx^3 - (3bx^2)\cdot(x - a)}{(bx^3)^2} \rightarrow y' = \frac{bx^3 - 3bx^3 + a3bx^2}{b^2x^6} \rightarrow = \frac{bx^3}{b^2x^6} - \frac{3bx^3}{b^2x^6} + \frac{3abx^2}{b^2x^6}$$

$$y' = \frac{1}{bx^3} - \frac{3}{bx^3} + \frac{3a}{bx^4} \rightarrow y' = \frac{1}{bx^3}(1 - 3 + \frac{3a}{x})$$

xii. $y = x(x+1)\cdot(x+2)$

$$y = (x^2 + x)\cdot(x+2) \rightarrow (x^3 + x^2 + 2x^2 + 2x)$$

$$y' = 3x^2 + 2x + 4x + 2 \rightarrow 3x^2 + 6x + 2$$

xiii. $y = (2x + 6)^3$

$$y' = 3(2x+6)^2\cdot(2) \rightarrow 6(2x+6)^2 \rightarrow 6(4x^2 + 24x + 36) \rightarrow 24x^2 + 144x + 216$$

xiv. $y = (x+5)^6$

$$y' = 6(x+5)^5 \; y' = 6(x+5)^5$$

xv. $y = (2x + 6)(5x - 7)$

$$y' = 2(5x - 7) + 5(2x + 6) \rightarrow 10x - 14 + 10x + 30 \rightarrow 20x + 16$$

xvi. $y = (2x + 6)^2.(5x - 7)^3$

$$y' = 2(2x+6)\cdot 2\cdot(5x-7)^3 + 3(5x-7)^2\cdot 5\cdot(2x+6)^2 \rightarrow (8x+24)\cdot(5x-7)^3 + 15(5x-7)^2(2x+6)^2$$

xvii. $y = -\sqrt{25 - x^2}$

xviii. $f(x) = 8\sqrt{3x^4 - 2x^2 + 1}$

$$\frac{df(x)}{dx} = 4(12x^3 - 4x)(3x^4 - 2x^2 + 1)^{-1/2}$$

xix. $f(x) = \frac{x^2 + 1}{3x^3 - 2x}$

$$\frac{df(x)}{dx} = \frac{(2x)(3x^3 - 2x) - (x^2 + 1)(9x^2 - 2)}{(3x^3 - 2x)^2}$$

xx. $f(x) = 4xe^{(x^2 + 4x)}$

$$\frac{df(x)}{dx} = (4e^{(x^2+4x)}) + (4x)(2x+4)e^{(x^2+4x)}$$